SM3 6.4: Compose Functions

Vocab: $(f \circ g)(x) = f\big(g(x)\big)$ The $\circ$ operator is a small, centered circle. It is said
            $(g \circ f)(x) = g\big(f(x)\big)$ "composed with" or "of", for short. It means to evaluate
            the left side function of the right side function.

Problems: Given $f(x) = 2x + 5$ and $g(x) = 3x - 2$, simplify the expressions:

1) $(f \circ g)(x)$ 2) $(g \circ f)(x)$

3) $(f \circ g)(2)$ 4) $(g \circ f)(0)$

5) $(f \circ f)(x)$ 6) $(g \circ g)(x)$

7) $(f \circ f)(9)$ 8) $(g \circ g)(-1)$

Given $f(x) = \{(1, 1), (2, 3), (4, 2), (3, 4), (5, 0)\}$ and $g(x) = \{(0, 1), (1, 2), (2, 3), (3, 4), (4, 5)\}$, simplify the expressions.

9) $(g \circ f)(x)$ 10) $(f \circ g)(x)$

11) $(g \circ f)(4)$ 12) $(f \circ g)(2)$

Given $c(x) = \{(1, 2), (2, 3), (3, 4), (4,1)\}$ and $d(x) = \{(1,3), (2,4), (3,1)\}$, simplify the expressions:

13) $(c \circ d)(x)$ 14) $(d \circ c)(x)$

15) $(c \circ d)(2)$ 16) $(d \circ c)(3)$

Given $r(x) = 4x$, $s(x) = x^2$, and $t(x) = x - 3$, simplify the expressions:

17) $(r \circ s)(x)$

18) $(s \circ t)(x)$

19) $(r \circ s \circ t)(x)$

20) $(r \circ t \circ s)(x)$

21) $(s \circ s \circ s)(x)$

22) $(t \circ r \circ s \circ t)(4)$

Express the function in the form $f \circ g$ by identifying what $g(x)$ and $f(x)$ are.

23) $F(x) = (x - 9)^5$

24) $F(x) = \sqrt{x} + 1$

25) $G(x) = \dfrac{x^2}{x^2 + 4}$

26) $G(x) = \dfrac{1}{x + 3}$

27) $H(x) = |1 - x^3|$

28) $H(x) = \sqrt{1 + \sqrt{x}}$

Application: You are an investment broker at Wytiaz Brokerage Firm with access to information about how different banks will pay for access to your clients' funds. Goliath National Bank (GNB) pays $52 if you invest your money with them for a year. First Brooklyn Savings Bank (FBSB) pays 5% of your investment if you invest your money with them for a year.

Write a function, $g(x)$, that models the money you could earn from the first bank (GNB) in one year:

   29) $g(x) =$

Write a function, $f(x)$, that models the money you could earn from the second bank (FBSB) in one year:

   30) $f(x) =$

Write a composition of functions that models the money you could earn by investing for a year in the second bank, then withdrawing your funds and investing in the first bank for a year:

   31)

32) An important client's daughter has $1000 to invest for two years. Mr. Wytiaz wants you to come up with the best plan for her yearly investments in order to maximize her money. Devise such a plan and write a sentence or two explaining why your plan is the best plan. Note that when writing to your boss, you should try and seem impressive and worthy of promotion!